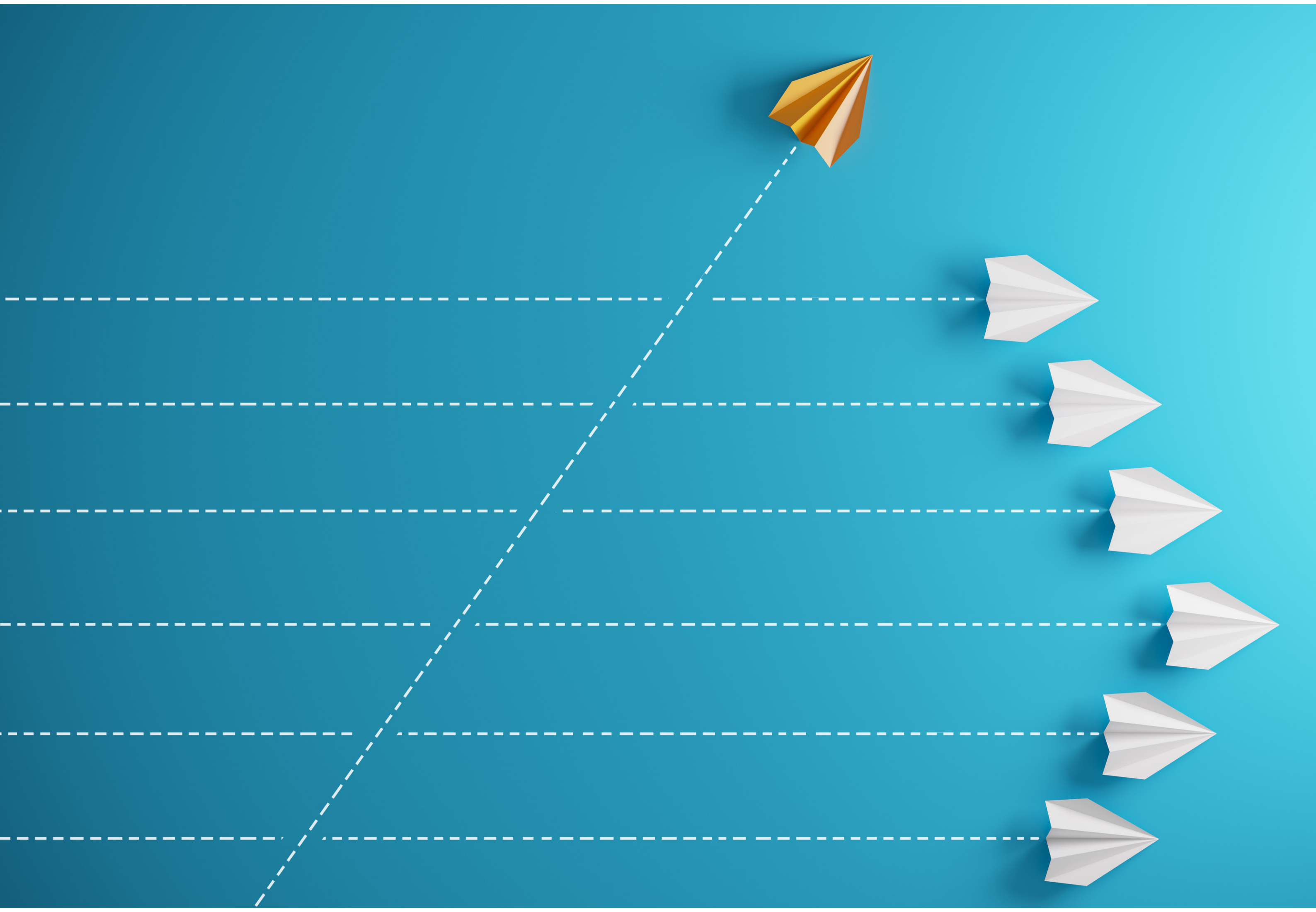
THE MATHEMATICAL ASSOCIATION OF VICTORIA

THE COMMON DENOMINATOR 3/21

LEARNERSHIP = SKILFUL LEARNING



INSIDE

James Anderson - Author, speaker, educator

Do you ever feel like you're dragging your students through the learning process? *You* set most of the challenges. *You* plan for and scaffold the learning. *You* find their mistakes, and then *you* show them how to correct them. It can seem like some students just coast along. They complete most of the work, but you worry they're not *really* challenging themselves.

One of my biggest concerns was that too many of my students weren't taking charge of their learning. They were passive learners. My students were learning the basic maths concepts and moving through the year levels. They were even getting decent grades and experiencing success in terms of achieving maths outcomes. But I often had the nagging feeling I wasn't preparing them well enough for a life of learning.

I now realise these students lacked what I've come to call *Learnership*.

Continued on page 4

FROM THE PRESIDENT

Michael O'Connor

WELCOME TO THE MAV BOARD 2021 TO 2022

The MAV AGM was held on Tuesday 25 of May. Unfortunately due to COVID restrictions that hit earlier that day, the event was moved to virtual at the last minute. We still had a great turnout of members, and a big thank you to all those dedicated supporters who attended. This year for the first time voting was electronic and available 21 days prior to the AGM. The MAV Board recognises that voting is an opportunity for greater interaction with members, increased transparency, and a wider say by members in the leadership of the Association. The voting process is now structured to meet these aims. In response to this change we had 8% of members vote, which is a great outcome as we usually have only those attending the meeting vote, which is previously around 1% of members. The outcome of the election sees the following people re-elected to the MAV Board for the period May 2021 to May 2023:

- Michael O'Connor, Casey Grammar School
- Louise Gray, Stitch Marketing
- Max Stephens, The University of Melbourne

MAV also welcomes the following two new members, elected to the Board for the first time:

- Kerryn Sandford, Heathmont Secondary College
- Nadia Abdelal, eXpanding Minds Maths

The following Directors continue their term until May 2022:

- Dan Cloney, ACER
- Kate Copping, The University of Melbourne
- Claire Delaney, Lalor Secondary College
- Michaela Epstein, Maths Teacher Circles
- Peter Karakoussis, Scotch College
- Kathryn Rodriguez, St Mary of the Cross Point Cook

It is great to see such a diverse range of professionals representing and supporting the MAV on the Board. You can read more about the Board including each person's biographies at www.mav.vic.edu.au/About-Us/MAV-Board.

NEW JOURNAL EDITOR

MAV welcomes Dr Angela Rogers as Editor of *Prime Number*.

The purpose of our primary journal, *Prime Number,* is to provide educators with support in their practice as professional teachers, to demonstrate and expose exemplary practice, new ideas and approaches, teaching strategies and pedagogy, providing practical tasks, and to show how research is translated into practice to improve teaching and learning. Angela is very well placed to do all of this. As Editor Angela will both write content, and work with other authors to help them publish articles in support of improving teaching and learning.

Angela brings a wealth of experience to the role of Editor, she is currently a lecturer in the School of Education at RMIT university in Melbourne. Angela loves helping pre-service teachers develop a love of, and confidence in, teaching numeracy.

Angela completed her PhD in Maths Education in 2014 where she developed an assessment tool for whole number place value. Angela has four children and loves nothing better than to play games and talk maths with them (or anyone who will listen!)

MAV members can look forward to the rich content and connections that Angela will make through the *Prime Number* journal.

If you would like to consider contributing an article, contact Angela at primenumber@mav.vic.edu.au.

THE COMMON DENOMINATOR
The MAV's magazine published for its members.

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UPCOMING MAV EVENTS

For more information and to reserve your place at any of the events below, visit www.mav.vic.edu.au.

EVENT	DATE	YEARS	PRESENTER
Conference: What's the glue that binds our mathematics curriculum together? Proficiencies, dispositions, content	12/7/21 Wangaratta	F-12	Various, including Dr Paul Swan, Andrew Lorimer-Derham and Helen Haralambous
Embedding the maths proficiencies in classroom practice	14/7/21 Virtual	7-10	Michaela Epstein
Number talks - what are they and how do they serve all learners?	15/7/21 Virtual	F-6	Alex Box
Visual manipulatives to assist understanding	20/7/21 Virtual	F-6	Tom Moore
Conference: Building articulate mathematical thinkers with the proficiencies	23/7/21 Horsham	F-12	Various, including Dr Sharyn Livy, Ellen Corovic and Thomas Moore
How can visuals prompt discussion that value and extend every learner?	29/7/21 Virtual	F-6	Alex Box
Learnership professional learning series	Aug-Oct Virtual	F-6	James Anderson and Jennifer Bowden
Formative assessment: The power of just in time assessment and instruction	5/8/21 Virtual	F-6	Brooke Brennan
Differentiating teaching: one lesson for all! - Part 1	10/8/21 Virtual	5-8	Michele Klooger
Data use for instruction: Beyond the spreadsheets and numbers	12/8/21 Virtual	F-6	Brooke Brennan
Differentiating teaching: one lesson for all! - Part 2	19/8/21 Virtual	5-8	Michele Klooger
Intervention: Prevention is better than cure - Part 1	26/8/21 Virtual	F-6	Paul Swan
Differentiating teaching: one lesson for all! - Part 3	31/8/21 Virtual	5-8	Michele Klooger
Intervention: Prevention is better than cure - Part 2	2/9/21 Virtual	F-6	Paul Swan and Narelle Rice
Unit 4 Networks (common traps and misconceptions that are seen on VCAA exams etc.)	8/9/21 Virtual	VCE	Jess Mount
Conference: New frontiers of learning	10/9/21	EY-6	Various
MAV Annual conference	2/12/21 and 3/12/21	EY - VCE	Various

MATHEMATICAL WRITER'S MASTERCLASS - TERM 4 2021

Would you like to meet occasionally with other teachers to support each other in writing classroom resources for students? The processes would be similar to the writing groups run in community libraries. Individuals bring some writing and share it with others. The goal is that, together, the group provides supportive advice to anyone who wants to write resources and activities, to help them take it to the next level.

The group will be led by Peter Sullivan and will initially run twice in Term 4. This opportunity is open to primary and secondary teachers.

If you are interested in participating, please email jbowden@mav.vic.edu.au with a brief EOI (100 words max) by 31 July.

Attendance at both sessions is required. Places are limited to 10 people for the trial.

Location: MAV, 61 Blyth Street, Brunswick

Dates: Thursday 21 October and Thursday 25 November

Time: 4.30 to 6pm

Cost: $15

LEARNERSHIP= SKILFUL LEARNING

James Anderson - Author, speaker, educator

CONT. FROM PAGE 1.

LEARNERSHIP: THE SKILL OF LEARNING

James Anderson

	CHALLENGE	HABITS OF MIND	MISTAKES	FEEDBACK	TIME & ENERGY
AGILE LEARNER	EMBRACES	CULTIVATES	DESIGNS	TAILORS	GROWING
INDEPENDENT LEARNER	TARGETS	DEVELOPS	USES	REQUESTS	STRIVING
DIRECTED LEARNER	ATTEMPTS	EXTENDS	CORRECTS	RESPONDS	PRODUCING
PERFORMANCE LEARNER	LIMITS	APPLIES	AVOIDS	SELECTS	PERFORMING
BEGINNER LEARNER	REDUCES	DESCRIBES	RECOGNISES	ACKNOWLEDGES	DOING
NON-LEARNER	AVOIDS	IGNORANT	IGNORES	DISREGARDS	WASTES

www.jamesanderson.com.au/learnership

Figure 1.

WE NEED QUALITY TEACHING

Through my work with schools, I know the feelings I've shared above are experienced by teachers all over the world. It's not that we aren't good teachers. We are! I believe that teachers in today's classrooms are amongst the most committed, best trained and best resourced we've ever had. But something has been missing from our practice.

Over the past 10 years, to improve student achievement, there has been an unprecedented focus on what quality teaching looks like. One outcome of this work is the *Australian Professional Standards for Teachers*, which clearly documents what quality teaching looks like.

I agree that we undoubtedly need skilful teachers who understand their students and how they learn. We need teachers who can build positive relationships, engage students in the learning process and provide purposeful instruction. But quality teaching on its own is not enough. In fact, we may waste a lot of our time and professional learning budget if we don't simultaneously focus on quality teaching and *quality learning*.

QUALITY TEACHING VERSUS QUALITY LEARNING

The difference between quality teaching and quality learning is the difference between what the teacher does in the classroom and what the student does.

For example, Standard 5 in the *Australian Professional Standards for Teachers* emphasises the importance of teachers giving appropriate feedback to students. Skilful teachers understand how to carefully craft formative feedback. Learnership, however, is about how students respond to that feedback. The best quality feedback in the world delivered by the best teacher in the world won't yield any benefit if the student ignores it.

In a similar way, Standard 3 highlights the importance of teachers setting challenging learning goals. A skilful teacher identifies a student's Learning Zone and challenges them appropriately. Learnership is about the student's attitude towards those challenges. Many students avoid challenging tasks, choosing to do easy tasks over difficult ones.

At the end of the day, student achievement isn't only about quality teaching.

There is another part to the student achievement equation:
Student Achievement = Quality Teaching x *Quality Learning*

QUALITY LEARNING FOR AND BY STUDENTS = LEARNERSHIP

Learnership is the skill of learning. It represents what quality learning looks like.

Think of Learnership in the same way you think of craftsman*ship* or leader*ship*. It denotes a skilful act. As students develop their Learnership, they engage in the process of learning in increasingly effective ways. Consequently, they can achieve more.

The Learnership Matrix is a bit like an 'Australian *Learning* Standards for *Students*', describing five key characteristics of learning. By identifying a learner's attitudes and responses to these five characteristics, we can define how skilfully they engage in the learning process.

Take a moment to reflect on the Learnership Matrix and identify how skilfully your students engage in the learning process. Like all skills, Learnership is developed over time.

While the Learnership Matrix is useful in identifying where students are at in developing the skill of learning, the real value of understanding Learnership is that it informs teachers how to help students become more skilful learners. We are not just teachers of maths; we are teachers of learning!

BECOMING AGILE LEARNERS

The most skilful learners are Agile Learners. These learners not only understand they are capable of growth (they have what we call a Growth Mindset), but they also understand how to achieve that growth. In many ways, they become their own teacher.

Agile Learners embrace challenges in the spirit of former US president John F. Kennedy, when he famously announced that America would put a man on the moon: 'We choose to go to the moon ... not because it is easy, *but because it is hard*.'

These students understand that challenges allow them to cultivate their Habits of Mind, giving them new, transferable skills and abilities they can apply in new and novel contexts. In mathematics and numeracy, this includes confidently applying the maths they understand to different situations, preferably related to real life. It also includes tackling mathematics based problem solving and selecting strategies that help them progress as required.

Agile Learners recognise and value the need to gather new information. They actively seek to fill the gap between the known and unknown by having specific learning goals in mind and identifying the information they need to reach them. These learners proactively request feedback and tailor it to their goals.

Perhaps most importantly, Agile Learners value their time and energy as limited resources and wisely choose to invest them in achieving growth. They understand it's not the *amount* of time and energy they spend that's important, but rather *how* that time and energy are spent. They are careful to avoid wasting time that could be invested in self-improvement.

A CLASSROOM OF SKILFUL LEARNERS

Imagine your classroom was filled with Agile Learners or even learners who have moved up just one level in the Learnership Matrix. How many frustrations described at the beginning of this article would reduce or disappear altogether? How much more would students achieve?

When we recognise that Student Achievement = Quality Teaching x *Quality Learning*, we draw attention to the importance of helping students become skilful learners and preparing them for a life of learning by developing their Learnership.

Think of the mathematics-based problems that could be solved or at least progressed if our students were Agile Learners – not just in the classroom but also beyond school as active members of society.

For further information about Learnership, visit www.jamesanderson.com.au/learnership.

LEARNERSHIP PD

In this series of workshops, you'll unpack the Learnership Matrix, identify the type of learners you have in your classroom, and learn how to guide your students in becoming more skilful learners.

As an experienced teacher you've honed your craft over many years. You know how to teach and you're good at it. But quality teaching isn't always enough. Learning is the product of what the teachers does and what the learner does. If our students aren't engaging effectively in the learning process, then even the best teachers face an uphill battle!

HOW DOES THE COURSE WORK?

This is not a program, and it's not extra work. This is about engaging in pedagogies in mathematics that focus on students becoming better learners.

Each fortnight you'll unpack a different aspect of learnership. This includes completing an online learning module, and a 90-minute webinar to consolidate the knowledge and skills you need to progress. You will also receive extra resources for extending your learning, to use in the mathematics classroom and to share experiences with colleagues.

This is a highly practical course. While you'll be given deep insights into the learning process, the focus is very much on what you do in your classroom to help students become better learners. Each topic includes tools, strategies, activities and techniques for you to apply immediately in your maths classroom.

While the strategies you learn are applicable across all learning areas, the specific examples and applications will be given in the context of teaching maths.

PRESENTERS

James Anderson and Jennifer Bowden.

MODULES

The course comprises six modules which are delivered fortnightly. Each session runs virtually from 4pm – 5.30pm.

Module 1: Introduction to learnership
Monday 2 August

Module 2: Growth mindset
Monday 16 August

Module 3: Challenges and mistakes
Monday 30 August

Module 4: Habits of mind
Monday 13 September

Module 5: Effort
Monday 11 October

Module 6: Developing learnership
Monday 25 October

For more information and to register for the workshops, visit www.mav.vic.edu.au/events.

PREPARING FOR VCE EXAMS

Fiona Latrobe, Mary Papp, Allason McNamara and Sue Garner

It's that time of year when VCE students are busy preparing for the upcoming exams. This article reveals some handy tips for each of Further Mathematics, Methods and Specialists - as well as common errors and misunderstandings made by students in the 2020 exams. Your VCE students may find this article useful.

FURTHER MATHEMATICS

Fiona Latrobe - Experienced teacher and long term Assessor Further Mathematics Exam

To succeed in Further Mathematics examinations students will need an in-depth understanding of concepts as well as an ability to apply these concepts. The very best students will have both manual and CAS skills and will use their CAS calculators selectively. Examinations cover material that is very basic through to very challenging and students should be encouraged to work at all levels of difficulty.

- Past examinations and examiner's reports are an essential resource as many questions reappear in a similar form over time.
- Students should also expect the examiners to include some questions that have not been seen before.
- Working to time is essential and students should include reading time practice when completing an entire examination.
- During reading time students should carefully read all questions, ensuring they clearly understand what is being asked and consider the best approach to each question. They should also look for sections of the examination where they will feel most confident to begin.

It is quite common for students to believe that they should use their CAS calculator in every question, but often students can work more efficiently without technology in certain circumstances. An example from the 2020 Exam 2 was question 1b in the core, where the median BMI in an ordered stemplot of 32 values was required. Students who recognised that the median was midway between the 16th and 17th values would have found the solution more efficiently than those who relied on the CAS calculator.

EXAM 1 TIPS

Exam 1 consists of multiple-choice questions.

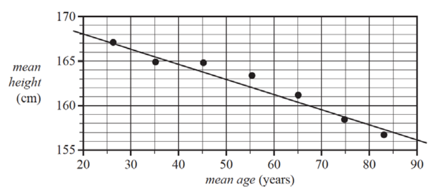
- Data analysis questions generally follow the order of the data coursework, starting with univariate data, then bivariate data and finishing with time series.
- The remaining sections of the paper tend to start with easier questions and finish with the most difficult questions.
- Students should familiarise themselves with the VCAA multiple choice answer sheet and remember that marks are not deducted for incorrect answers, so an attempt should be recorded for each question.
- It is often a good idea when answering multiple choice to attempt each question, without reference to the available options, and then to check the solution found against those available. This can assist students in avoiding distractors and make sure that the focus is on what is required in the question.

EXAM 2 TIPS

- Questions require legibly written answers. Sentences are usually not required unless a description or explanation is asked for, so students should make their answers as succinct as possible using dot points, if necessary, to make their answers clear.
- Most questions are worth one mark requiring only an answer, but some questions will be worth two marks. Students should always show some working in two-mark questions. Relevant working may be awarded one of the two marks if the final answer is incorrect.
- Some questions require students to demonstrate solutions. In these questions students should be careful to show as many steps as required and they must work from the original problem towards the demonstrated solution rather than just verifying the required solution
- Careful checking after the answer is written is important.
- Reading the question again to ensure that their solution is complete, that it has answered what has been asked.
- Check details such as the required level of accuracy will avoid simple mistakes.
- Careful rounding is always important. Writing an unrounded version first and then writing the rounded version will limit loss of marks and students must be particularly careful to write monetary answers correct to the nearest cent, unless there is a different instruction in the question.

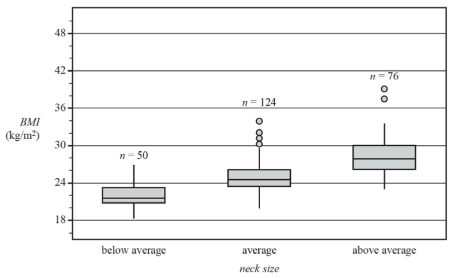
COMMON ERRORS MADE BY STUDENTS (2020 EXAM 2)

- Significant numbers of students wrote monetary answers correct to the nearest dollar, resulting in significant loss of marks. An answer with less detail than the required solution is an absolute error and is not considered a rounding error, so marks are deducted every time this occurs.
- Students should also avoid answering questions by stating unwanted extra information. For example, Q1a in Data analysis where students were asked for the shape of a stem plot. The correct answer was 'positively skewed', but many students incorrectly included that there were possible outliers, making their overall answer incorrect.
- Another example was data analysis Q6b where the strength and direction of the association shown below was required.



Many students correctly stated that the association was strong and negative, but then added either 'linear' or ' non-linear'. As students were not asked for the form of the association, assessors could not be certain that the student understood what strength and direction meant, so the correct answer was negated by the extra information.

- Data analysis Q3c, where students were asked to find the number of men with a BMI over 30 from the parallel boxplots shown here:



Data: RW Johnson, 'Fitting percentage of body fat to simple body measurements', *Journal of Statistics Education*, 4:1, 1996, <https://doi.org/10.1080/10691898.1996.11910505>

There were no men with a BMI above 30 in the below average group, four (all outliers) in the average group and 19 (25% of the 76 men) in the above average group, making a total of 23. Many students incorrectly added the two outliers from the above average group to the 19 in the quartile.

EFFICIENT USE OF THE CAS

Examples have been given above of efficient solutions that do not use CAS calculators but knowing how to use the CAS effectively is very important in this technology active subject. The use of the Financial function on the CAS calculator should be used by students and if a question has two or more steps, entries into this menu are considered appropriate working for Exam 2.

EXAMPLE OF A QUESTION NOT ANSWERED WELL BY STUDENTS (2020 EXAM 1)

Q30: Twenty years ago, Hector invested a sum of money in an account earning interest at the rate of 3.2% per annum, compounding monthly. After 10 years, he made a one-off extra payment of $10 000 to the account. For the next 10 years, the account earned interest at the rate of 2.8% per annum, compounding monthly. The balance of his account today is $686 904.09 The sum of money Hector originally invested is closest to

a. $355 000

b. $370 000

c. $377 000

d. $384 000

e. $385 000

Use of the Financial function was essential to answer this question and students had to recognise the steps required:

Step 1: Finding the balance ten years before the end

N = 120   I = 2.8%   PV = ?
PMT (or Pmt) = 0 FV = 686 904.09
P/Y and C/Y (or PpY and CpY) = 12

The FV is positive as the money moves towards Hector at the end of the investment. The PV is found to be -$519 320.30. This value is negative as Hector put that money away from himself at the start of the investment.

Step 2: Subtracting the $10 000 payment made ten years ago

$519 320.30 - $10 000 = $509 320.30

Step 3: Finding the balance ten years before the deposit of $10 000

N = 120   I = 3.2%   PV = ?
PMT (or Pmt) = 0   FV = 509 320.30
P/Y and C/Y (or PpY and CpY) = 12

Again, FV is positive as the money (temporarily) came to Hector in the middle of the investment. The PV is calculated to be $369 999.78, which was closest to $370 000, answer B.

Similar questions have appeared in Exam 2 for two marks and students have incorrectly rounded at the end but have neglected to show any working from CAS along the way. An incorrect answer with no supporting working shown will receive no marks.

MATHEMATICAL METHODS

Mary Papp (experienced teacher and long term assessor for Mathematical Methods Exam 1) and Allason McNamara (experienced teacher and long term assessor for Mathematical Methods Exam 2)

EXAM 1

- Questions worth more than one mark require reasoning. Ensure that your methodology is clearly and **legibly** communicated. Best practice is to work down the page and make your final answer obvious.
- Exam 1 highlights algebraic manipulation. Practice solving equations including, algebraic fractions, quadratic, logarithmic and trigonometric equations.
- Be especially vigilant with negative signs and incorrect cancellations. Take special care with brackets.
- Know the exact trigonometric ratios, index laws and logarithm laws.
- Quoting a rule is not adequate to gain a method mark. You need to apply the rule to the information given in the question itself. (Put in values etc.).
- Check that you have answered the actual question. Was a specific format for the answer required? Did you take account of a restricted domain? Was a rule and domain or range requested? Do you have the correct notation?
- 'Show that' questions are generally poorly handled. The answer is given so marks for this style of question are awarded for a reasoned argument as to how the answer is obtained.

COMMON ERRORS MADE BY STUDENTS (2020 EXAM 1)

Q1b. Many students did apply the chain rule but were sloppy with their final answer, dropping brackets entirely. The correct answer to the derivative was $(2x-1)e^{x^2-x+3}$
It was **not** $2x-1e^{x^2-x+3}$

Q2. This was a probability question. Use of a Venn diagram or Karnaugh map was of advantage and was a viable method of showing your interpretation and reasoning.

Also many students had difficulty in transposing the equation $\frac{n}{m+n}-\frac{1}{m+n}=\frac{1}{20}$ in order to make m the subject (i.e. $m=\ldots$)

Q3. This question involved setting up simultaneous equations and solving. A common error was to state $\tan^{-1}(-1)=\frac{3\pi}{4}$ however $\frac{3\pi}{4}$ was outside the given domain.

Also, many could not correctly evaluate $\tan^{-1}\left(\sqrt{3}\right)$.

Q4. Solving this log equation produced 2 possible answers, but one answer was not valid as it led to a solution that is impossible (i.e. the log of a negative number). Check that your answer is valid!

PREPARING FOR VCE EXAMS (CONT.)

Q5b. This was a conditional probability question. Do more than state the rule. Also take care to answer in the format required – this required some manipulation with index laws and common factors.

Q6. This question involved sketching an inverse. Take care with curvature. Use a **pencil**, not pen for graphs. While students correctly identified the end point at (1,2), they did not plot it correctly on the given grid. The remainder of the question involved areas. This meant evaluating definite integrals involving fractions and surds. Students who were 'all over the place' in setting out or tried to do too many steps in one line generally made arithmetic errors.

Q7. Failure to answer the specific question was the main error. Part b(i) asked for the slope (gradient) of the line connecting two points, while part b(ii) asked for the slope at a point. They are different things. Failure to recognise this caused errors with the rest of the problem.

Q8. There were errors with differentiation or evaluations involving 'e'. Part b was a 'show that'. Students should give a step-by-step explanation. This question also involved definite integrals. Errors in evaluation (arithmetic), and dropping negative signs were common.

Practice, Practice, Practice

EXAM 2

- Make sure you reread questions.
- If a rule is required, make sure you give a rule not just an expression.
- Do not assume steps in 'show that' questions.
- Work to more decimal places than what is required in the answer.
- Give exact answers unless otherwise stated.

COMMON MISUNDERSTANDINGS FROM 2020 EXAM 2

In Question 5c students were asked for the nature of a graph. The graph was a horizontal line. Some students thought they had to describe a stationary point and gave local minimum or local maximum as their answer.

REASONS STUDENTS LOST MARKS

- In the multiple choice section of the Exam 2, students were losing marks in questions that required more than one step of working. For example, in Question 16 many students found the value of m and stopped, giving Option C as their answer. Students were asked to find the maximum area of the triangle which was Option D. Similar scenarios occurred in Section B Question 1c(ii), 2b, 2c and 4d(ii).
- Make sure you write your answers correctly when transcribing from the calculator. In Question 4c $2\left(1-2p^2\right)e^{-p^2+1}$ was incorrectly written as $2\left(1-2p^2\right)e^{-p^2}+1$ and in 4e(ii), e^{n^2-8} was often written as $e^{n^2}-8$.
- Rules were required in Questions 1c(i), 4e(i) and 4e (ii), for example in Question 4e(i) some students wrote $2e^{1-n^2}x$ instead of a rule $y_1 = 2e^{1-n^2}x$.
- In Questions 4a and 4c the questions asked for the gradient not the equation of the tangent. Make sure you answer the specific question.
- In Question 5a some students were not using brackets correctly. $a^3 - a = (3a^2 - 1)(a - b)$ was incorrectly written as $a^3 - a = 3a^2 - 1(a - b)$. Brackets were used incorrectly in questions 1b, 1e(ii), 2d, 2f, 4c, 4e(ii), 4e(iii) and 5a.
- Exact answers were required in Question 1c(ii), 1e(i), 2b, 3f, 5b, and 5e. Some students gave decimal approximations.

An example of a question that caused difficulty and was not well answered

Question 19 of the multiple choice section of Exam 2 was not answered well. Only 15% of students managed to get this correct. One strategy is to use specific values to try to find a pattern and then attempt to generalise as shown here.

$$q \sim \mathrm{Bi}\left(20, \frac{5}{6}\right),\ p \sim \mathrm{Bi}\left(20, \frac{1}{6}\right)$$

Examples

$$q(19) = \binom{20}{19}\left(\frac{5}{6}\right)^{19}\left(\frac{1}{6}\right) = p(1) = \binom{20}{1}\left(\frac{1}{6}\right)\left(\frac{5}{6}\right)^{19}$$

$$q(18) = \binom{20}{18}\left(\frac{5}{6}\right)^{18}\left(\frac{1}{6}\right)^{2} = p(2) = \binom{20}{2}\left(\frac{1}{6}\right)^{2}\left(\frac{5}{6}\right)^{18}$$

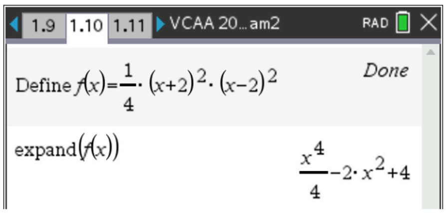
In general

$$q(w) = \binom{20}{w}\left(\frac{5}{6}\right)^{w}\left(\frac{1}{6}\right)^{20-w} = p(20-w) = \binom{20}{20-w}\left(\frac{1}{6}\right)^{20-w}\left(\frac{5}{6}\right)^{w}$$

$$q(w) = p(20-w)$$

EFFICIENT USE OF THE CAS

- Use your technology efficiently.
- Define functions at the start of the question. Question 1b. required students to expand a function. This can be easily done using technology. There is no need to expand it by-hand as it is time consuming and often algebraic errors are made.



SPECIALIST MATHEMATICS

Sue Garner (experienced teacher and long term assessor for both Specialist Mathematics and Mathematical Methods Exams)

Firstly, remember that 2020 was a unique year in which both students and teachers had a mixed year due to COVID-19. The changes for 2020 in Specialist Mathematics meant that the whole Area of Study: AoS6 Probability and statistics, was deleted. This means that past exams 2016-2019 will have questions from AoS6 but that the 2020 exam will not. Some general tips for success are:

- Completing and marking as many past exams as you can, is the best preparation for exams.
- Past exams are freely available on the VCAA website.
- Be systematic. Make the decision as to how many you think you can manage to complete. Because of their style and content, past exams come in three sections 2006-2009, 2010-2015, 2016-2020. Completing all of these exams (2 per year) totals to 30 papers as well as the recent year's NHT papers. (note that 2020 did not have NHT papers). A good suggestion is to select a starting point and work forwards

so that you reserve the most recent papers, 2020 Exams 1 and 2, to do for your last day or night's study session before the exams, (including an AoS6 section from a previous year).

- Carefully read the Examiners Reports either after or during your practice of that paper.
- Most recent years will be the best ones to concentrate on. Many of you will have done the Maths Methods exams on the previous two days, so pace yourself well.
- Commercially written trial papers are also an excellent source of practice.

EXAM 1

Exam 1 is often a relief where, if you know your material, you should be able to work through it using the best of your knowledge and preparation. There is nothing like being in reading time and recognising every question type. Be alert to common mistakes:

- Not showing working steps in 2 and 3 mark questions; the assessors will not be impressed with you if you want to show you can do a 3 mark question in your head.
- 'Fudging' proofs where you get to the required ending with dodgy algebra.
- Making arithmetic mistakes, especially fractions and negative numbers.
- Assuming all questions in Mechanics are using constant acceleration.
- Assuming all questions in Kinematics are in terms of t.
- Trying to solve a DE when the question asks you to 'verify'.
- Assuming all integration can be solved using change of variables.
- Not labelling graphs with equations of asymptotes and axial intercepts.
- Finding an exact answer and then decimalising it when not specifically asked for, not even using approximately equal ($\approx$).
- Giving θ if the question asks for $\cos(\theta)$.

Be confident of your algebra and graphs and if you make a mistake, don't erase, but just cross out and do again in a space available. If this space isn't available make sure you draw the examiners' attention that you have attempted it perhaps on a spare or the last page. Sketch your graphs in pencil and watch your concavity by perhaps plotting another point to carefully sketch the shape. Label endpoints with coordinates.

There will be some difficult algebra and plenty of practice of past exams will only help this, but don't leave blank spaces! Some work is better than none, and you may even be correct enough for 1 mark.
Try sketching a graph.

EXAM 2

Exam 2 will always be a difficult paper. The first decision is whether to attempt multiple choice section or the Extended Answer section first. It is disappointing if you take too long on multiple choice and turn to the last long question and realise you know it well and have run out of time. Make this decision before your practice of at least the previous 4 years of exams. Allow at most 40 minutes for the multiple choice section, and expect some of these questions to require deep thought and complicated working.

- Be aware when your calculator needs to be in Complex or Real form, Decimal or Standard form, Radian or Decimal.
- Use unrounded decimals throughout the question even if you have had to round off in an earlier sub-section.
- Do not write calculator syntax.
- Do not write what you would have done if you had time, there are no marks for this!
- Make sure you label all forces in Force diagrams, even if not asked for.
- Answer every part of the question for example; don't leave off the y-value, the unit vector, the magnitude of a force, the constant of integration, the tildes, the dx.
- Don't oversimplify expression when this is not required. Some students go on and on simplifying trig expressions when not required, making mistakes and losing marks on the way.

In preparation for the Exam 2, refine your bound notes, but there is no need to rewrite them. Your notes should evolve and build as the year progresses; this means you will remember easily which is which section when you need it. As you go through your past papers, add into your notes any gaps in your knowledge, good examples of questions you have come across and reminders you think you will need. A summary and index at the front or back will help. Colour code corners of pages if that helps you. You may be surprised you won't look at your notes during the exam as much as you expect.

COMMON MISUNDERSTANDINGS FROM 2020 EXAM 1

Q4: This was a difficult question found early in the paper. The best answers were from students who sketched a graph of the reciprocal absolute value function $\frac{1}{|x-4|}$ as well as the linear function $3-x$.
This made it clear there would be only 1 point of intersection. Those students who attempted the question with just an algebraic approach were not able to solve the inequation accurately.

Q9: The advanced algebra in this question confused many students. In both parts a and b, many students were unable to recognise and manipulate perfect squares. There was also confusion with brackets and if the square sits inside or outside the bracket: either $(1-t^2)$ or $(1-t)^2$. It is vital to realise this isn't specifically a Specialist Maths skill, but rather one of accurately written down algebra. Question 9a, to find the arc length, required students to recognise a perfect square. In these questions, the perfect square will always be there, so be persistent in your algebra.

$$\sqrt{\frac{1}{(1-t)^2}+\frac{1}{2(1-t^2)}+\frac{1}{16(1-t)^2}}$$

becomes the perfect square as required $\sqrt{\left(\frac{1}{(1+t)}+\frac{1}{4(1-t)}\right)^2}$, giving an expression that will easily find the arc length:

$$\int_0^{\frac{1}{2}}\left(\frac{1}{(1+t)}+\frac{1}{4(1-t)}\right)dt$$

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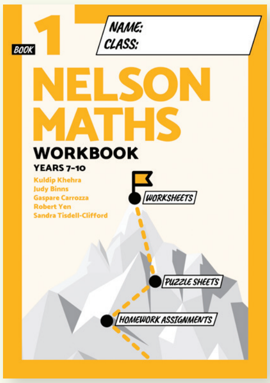

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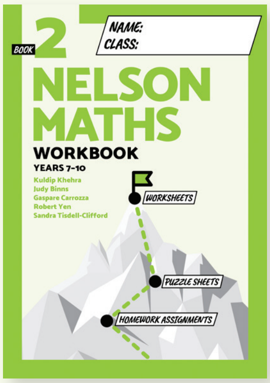

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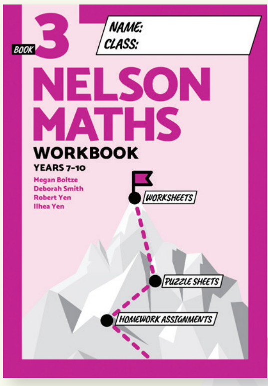

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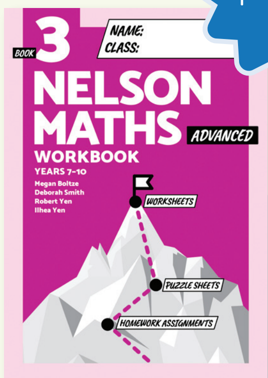

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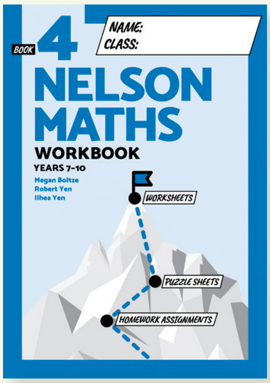

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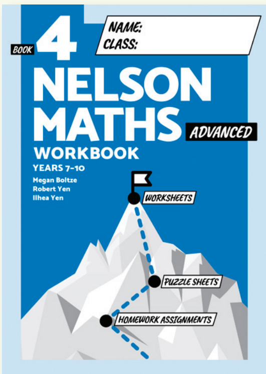

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PREPARING FOR VCE EXAMS (CONT.)

Other weaknesses include:

- Efficiently evaluating integrals using appropriate substitutions.
- Solving quadratic equations (Q4 and Q5a).
- Knowledge of exact values arising from trigonometric expressions.
- Simplifying arithmetic and algebraic expression.
- Brackets wrongly placed or not completed.
- Simplification of logarithm statements.
- Not actually using the formula sheet.

AREAS OF STRENGTH FROM 2020 EXAM 1 INCLUDED

- Resolving forces parallel and perpendicular to a plane (Q1).
- Expressing a complex number in Polar form and using de Moivre's theorem (Q3).
- Using calculus to show a given result (Q6a and 7a).
- Sketching of suitable graphs (Q6c).
- Understanding of area (Q7b).
- Recognising the correct form to use in a partial fraction problem (Q8).

COMMON MISUNDERSTANDINGS FROM 2020 EXAM 2

- Not reading and responding to all aspects of questions.
- Misunderstanding the distinction between velocity and speed.
- Not justifying an answer when required.
- Noting when a mechanics problem changes from constant acceleration to acceleration in terms of other variables.
- Answers were generally given in the required forms; however, there were indications in Section B that students did not always correctly read and respond to questions, particularly where there was more than one aspect to the question.
- Q4b(i) and 5b(i) where students needed to show that a given result was reached. Steps needed to be clearly and logically set out to obtain full marks.
- Q1b(i), not continuing the question to give the equation of the tangent to the path, after correctly stating the required derivative.
- Q2e not stating the radius despite indications in working that students had otherwise solved the problem.
- Not all required details on graphs were correctly given or values rounded appropriately.

AREAS OF STRENGTH FROM 2020 EXAM 2

- Determining cartesian forms of parametrically defined curves.
- Expressing relations in the complex plane in cartesian form.
- Sketching the curve of a given function.
- Use of CAS technology.

REASONS STUDENTS LOST MARKS IN EXAM 2

Multiple choice

- Q2: only 42% were able to use the transformations for $g(x) = \cos^{-1}(x)$
- Q4: only 28% considered the correct range for $f(g(x))$
- Q7: only 26% were able to choose option D by considering the given information that $b < 0$.
- Q8: 34% were able to manipulate the algebra in $(x + yi)^{14}$
- Q9: 35% could read slope fields.
- Q15: 38% correct: students needed to use $\sum \underset{\sim}{F} = m\underset{\sim}{a}$ and antidifferentiate $\underset{\sim}{a}$ twice to find the position vector.
- Q20: spring balance questions often cause problems for students and only 43% got this correct. The reading on the spring balance is the magnitude of the upwards force on the object.

Extended answer

Question 1

- Remember to use dt at the end of an integral.
- Lost marks by rounding to fewer than 3 decimal places
- Many students gave the (scalar) magnitude of the velocity rather than the required velocity.
- Made the mistake of using dx in Q1b(ii). rather than dt.

Question 2

- Although, generally well done students mixed up lines and rays.
- Q2d(ii), while a high proportion of students gave the correct rule, many didn't fully describe the function as they didn't include the domain. More a Maths Methods technique that is assumed.
- Students struggled with Q2e. Some were set up cartesian or complex equations, fewer were able to proceed further. Students familiar with their CAS were able to use it effectively. Some students correctly found z_c but did not also state the radius.

Question 3

- Students fell into the common trap of adding an extra asymptote in Q3b.
- Lost marks either for sketching a poor shape in the second quadrant or for incorrectly labelling points of inflection, including having the x-value of the left-most point of inflection rounded to 0.58 instead of 0.59.
- Q3e(ii), very few students were able to distinguish between even and odd values of n when considering multiple points of inflection.

Question 4

- Confused maximum speed with maximum velocity.
- Graphs were not drawn with sufficient accuracy and rounding of decimals.
- Questions were not answered completely to the end.

Question 5

- Students simplified answers incorrectly and didn't consider suitable values of k.
- Q5e only 21% were able to successfully work to the end. Many students didn't attempt this question. Of those who did, errors included using constant acceleration formulas, failing to appreciate that the masses were no longer connected and inconsistencies in signs and directions.

The 2021 exams will be written for you to show what knowledge you have, not to trick you, so go in ready to show how much you have learned this year. Good luck!

STIMULATING THINKING

Education consultants, Mathematical Association of Victoria

A picture sparks 1000 maths concepts! Use this picture as a prompt to stimulate thinking. If you have other ideas for investigations or lessons that could stem from the ideas here, add them to the conversation on our social channels. You can find us on Facebook, @mathematicalassociationofvictoria and on Twitter, @mav_info.

EARLY YEARS - YEAR 2

- What does this image make you wonder? Use mathematical language to describe what you see.
- Estimate how many staircases there are in the photo? How could you check your guess without counting each staircase?
- Name five 2D different shapes you can see in the picture. Compare you list with a friend; have they named any shapes that are different to yours?
- The building is extremely diverse. Can you find any two parts of the building that are the same? Use mathematical vocab to explain the similarities and differences.
- Who could live in this labyrinth? What size would they be to climb the stairs, walk through the doors and look out of the windows.
- Draw your own 2D version of a vertical labyrinth that has only one path to get from the bottom to the top.
- Of the five 2D shapes you found, can you now categorise them, and explain why you put them in the categories that you did?
- If this were a life size structure how long do you think it would take to walk around the outside of the whole structure?
- Can you describe where each of the towers is located?
- What is the likelihood that one family could live in this building?

YEARS 3 AND 4

- Name five 3D different shapes you can see in the picture. Compare you list with a friend; have they named any shapes that are different to yours?
- Estimate how many stairs there are in the photo? How could you check you guess without counting each stair?
- The vertical labyrinth appears to have no beginning and no end. Choose a starting point at the bottom of the labyrinth and give directions, using mathematical language to get to the top.
- Estimate and justify how many people might be living in this building? What is a number that is too high/too low?
- There are many square pyramids in this image. Describe the features of a square pyramid and explained how it is different to a square prism.
- Find five examples of symmetry in the labyrinth. Draw and describe these to a classmate.
- There are many horizontal, vertical, and diagonal lines in the labyrinth. Write a definition for horizontal, vertical and diagonal.
- If this were a life size structure estimate the height? Describe how you worked this out.
- If you walked up each of those staircases how long do you think it would take you?
- There are windows, doorways, staircases and towers in this building. What fraction of the total are each of these?
- Make a data display showing the number of each of these aspects of the structure. Can you also display this data in another way?

YEARS 5 AND BEYOND

- Create a bird's eye view map of this vertical labyrinth. Ensure you correctly apply scales and legends to your map.
- The majority of angles in the labyrinth are right angles or 45 degrees, can you find and name any other angles.
- MC Escher played with reflection and rotations to create his art. This labyrinth is very similar to his work. Find five examples of reflections and rotations in this image.
- The building is blue, but one side appears yellow as the sun is shining on it. Draw a picture or create a simple model to show where the sun would be in relation to the building for this image to be correct. Should there be a shadow?
- Only two sides of this labyrinth are visible. Draw a picture of one of the hidden sides.
- If this were a life size structure estimate the area and volume of the whole structure? Describe how you worked this out.
- Write a set of instructions to a specific location in the labyrinth. See if a partner can follow them.
- There appears to be many patterns in this structure. Choose one pattern and draw a continuation of the pattern.
- Using a scale from 0-1 indicate how likely it would be for the following to occur: walk up all sets of stairs in less than 20 minutes, to walk around the whole structure in less than 10 minutes, to go and look out every window in less than 30 minutes?
- Create and draw your own labyrinth.

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FOOTY: IT'S MORE THAN A GAME

Jennifer Bowden - Mathematics education consultant, MAV

The benefits of playing a good board or card game have been widely researched and celebrated. Not only are the obvious advantages of learning important content, skills, and strategies but the interpersonal and social skills are also widely acclaimed.

Over the holidays my family discovered the game of Mahjong. This is more than a game, its an intense battle of strategy, knowledge of patterns, game customs and language. Hours of sophisticated strategic game play went by, we were deep in its infinite rules and possibilities. The level of mathematical content and strategy in this face paced game keeps my family – aged 8 to 70 – in a heated battle, it was brutal and it was fun!

My family has also recently discovered and have started to enjoy the battle of the game of *Footy Feud*. This is an extremely well-designed card game that expands the rules of the game of Australian Rules Football to challenge the players mathematical fluency along with using strategies that encompass the other three proficiencies and engages a child's productive disposition towards mathematics.

As a family of Victorian's who love our footy, we are quite fluent when calculating a team's score based around goals and behinds. If you have managed to avoid the AFL, a goal is worth six points and a behind is worth one.

Over the years, I have enjoyed challenging students I've taught and now my own children with problems and tasks based around footy scores, it's a great way to see students move from additive to multiplicative thinking, *Footy Feud* goes a step further. It is based on students trying to create the highest score from their cards starting with the *goal* and *point* cards.

The challenge goes further with additional cards such as *tackle* (take an opponent's card and add it to your deck), *sharp shooter* (turns all points into goals), *full forward* (doubles your goals) and the *full back* (take twelve points off any opponent). The possibilities are endless, or are they? That's a mathematical task worth exploring.

The game is fast paced and played over four rounds, much like the four quarters of a real footy game.

Each game takes approximately five minutes. I like to have a mini-white board handy to record scores. Incidentally, the recording of scores and strategies in the games can become an excellent basis for number talks. *Footy Feud* is an excellent warm up activity that can be taken much further to look at intricate aspects of multiplicative thinking, pattern and algebra concepts.

Whilst the game has obvious use as a warm-up task, there are many wonderful mathematical investigations that could be implemented into the classroom.

- What is the highest possible score in a game?
- What is the lowest possible score in a quarter?
- How is a game altered if we remove the *tackle* cards?
- What if we changed the value of *goals* and *points*?
- Jen has 2 x *goals*, 3 x *behinds*, 1 x *full forward* , 1 x *sharp shooter*. What is her score? Compare your recording of you working out with a classmate.

It's not quite as sophisticated as Mahjong but it's certainly a lot more convenient and accessible.

Footy Feud is available from the MAVshop www. mav.vic.edu.au/mav-shop.

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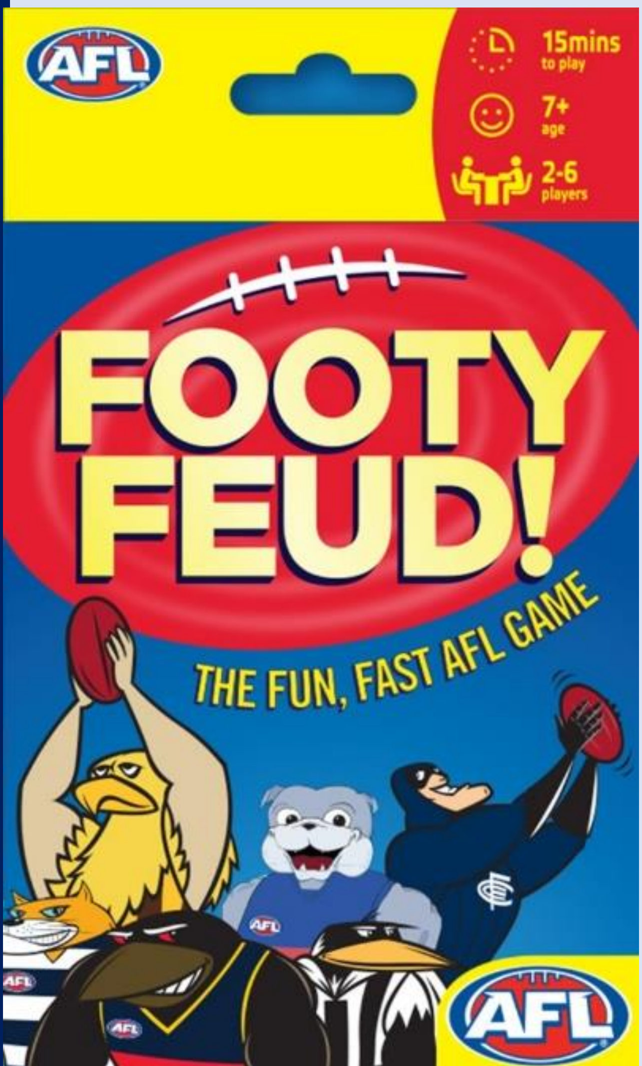

- ✓ *Each card is worth a different footy score while rare FEUD cards act as game changers. Players score in goals and behinds like a footy game*
- ✓ *Players play 4 hands, representing 4 quarters. Whoever has the highest score at the end of the game wins!*
- ✓ *"Footy Feud has now become a family favourite and the educator in me is very happy to see my family from 8 to 80 playing the game and flexing their mathematical muscles" Jen Bowden*

For more information, visit www.footyfeud.com

VCE SACS: POLITICS

Andrew Stewart

Formal political institutions such as Australian Parliaments, or the equivalent in other countries, offer interesting statistical investigation opportunities.

The Data Analysis section of MAV Trial Exam 2 2018 contained questions with a political background which were based on data from a variety of sources. In brackets under each question heading below, I have briefly summarised the question contents. For each question, I have suggested alternative activities, indicated the sources (and alternatives available) of the data used, as well as some guidance as to how the original questions were developed. It would be an advantage to have a copy of the Trial Paper on hand for reference.

QUESTION ONE (UNIVARIATE ANALYSIS)

(Stem plot of Australian Prime Ministers, analysis, parallel boxplot with New Zealand Prime Minister data)

The following lists show the age in years, correct to the nearest whole year, at which Prime Ministers first took office for Australia (top) and New Zealand (bottom) from 1901 to date.

52, 47, 37, 59, 46, 53, 53, 40, 53, 53, 59, 44, 47, 57, 55, 60, 57, 68, 56, 63, 56, 45, 52, 48, 57, 50, 49, 56, 61, 50

48, 50, 50, 59, 56, 74, 47, 61, 64, 56, 56, 54, 76, 60, 50, 47, 54, 42, 47, 42, 55, 46, 50, 47, 55, 37

Suggested tasks

(a) Graph these data sets in a way that would assist the answering of part (b).

(b) Complete tables containing the five-number summary, plus mode, (range?), mean and standard deviation, for these data sets.

(c) Construct a fully labelled parallel boxplot showing both the Australian Prime Minister data and the New Zealand Prime Minister data.

(d)(i) Chris Watson was just 37 when he became Australia's Prime Minister in 1904. John McEwen was 68 when he became Australia's Prime minister in 1968. Explain why these ages are **not** outliers to this data set.

(ii) Francis Bell was 74 when he became New Zealand's Prime Minister in 1925. Walter Nash was 76 when he became New Zealand's Prime minister in 1957. Explain why these ages **are** outliers to this data set.

(e) Are the z-scores for the politicians' ages in part (d) a good indicator for outlier or otherwise?

(f) Using the information from the boxplots, explain why the age of first taking office is associated with the country. Quote the values of appropriate statistics in your response.

These lists were restricted to Prime Ministers in office from 1901 to date, to give a comparison over the same time period. Technically, in New Zealand, Prime Ministers were only designated as such from 1907 to date, which would mean deleting the first two ages in the list. The references give the exact date of assuming office but do not give birth details, which had to be tracked down individually (usually from Wikipedia). As an alternative, there are lists available (via Wikipedia) of Australian State Premiers, or the leaders of many countries (46 US Presidents or many British Prime Ministers), and these could be compared to one another similarly to above.

QUESTION TWO (UNIVARIATE ANALYSIS, LOG GRAPH)

(A log graph of frequency against $\log_{10}$(area in square km) for all federal electorates with analysis)

A good challenge for students would be to construct a plot of frequency versus $\log_{10}$(area in square km) of all 150 Australian federal electorates, using a data sheet constructed from the given reference. (The answer is shown in the Trial Paper). A smaller (easier) task would do the same for all electorates (federal or state) in a particular state or territory.

An alternative challenge would be to construct a log graph of the time in office of Australian (or New Zealand) Prime Ministers, as these range from a few weeks to more than a decade.

QUESTION THREE (BIVARIATE ANALYSIS)

(Graph of *thousands of people per MP* versus *population in millions*, with analysis)

This question was originally constructed on a spreadsheet using data from a Wikipedia page 'List of legislatures by number of members', which lists countries, number of members and population per member among other interesting information. The graph in the Trial Paper question was based on countries with more than 100 members in the lower house, and, from a large list, was reduced to just the sixteen values provided.

An interesting feature to examine would be to examine populations of countries with around 150 lower house members, or to look at the number of members in countries with around 26 million population, to compare to Australia's situation.

As one alternative, students could be given all/most of the data from this webpage in a data sheet, and asked to randomly select a certain number of countries to analyse the possible existence of this relationship.

As another alternative, use the data for the 50 American states where House of Representative (or equivalent) numbers range from 400 members in New Hampshire for a population under 1.5 million to 31 members in Texas for a population of about 25 million.

To keep this activity with an Australian flavour, compare the information for the Australian States and Territories' Parliaments/Assemblies, or local government (councils and shires).

QUESTION FOUR (BIVARIATE ANALYSIS WITH TRANSFORMATION)

(Graph of *thousands of people per MP* versus *population in millions*, with transformation and analysis)

This graph, requiring transformation, was selected from the European Union members (same reference as Question Three). On my spreadsheet, I started with a large number of data points, and selectively deleted data to achieve the shape (and hence transformation required) and number of points desired.

By selectively deleting data values from a large list, it may be possible to construct several graphs, each requiring a different transformation.

QUESTION FIVE (TIME SERIES)

(Time series graphs with analysis)

Finding time series information about political situations (approval ratings of parties or their leaders) was difficult. There was a Common Assessment Task (CAT) back in 1996 with voting data from around 1990. Unless you are prepared to search through a large number of newspaper (or similar) sources, it may be quicker to make the data up.

For the time series graphs in this Trial Exam question, I created several spreadsheets showing both the data and its analysis, and changed the key data values until I obtained the results I was seeking.

ASSESSMENT

I always design SAC 1 so each student is given, or can generate, their own individual data set. This was to eliminate, as much as possible, students assisting one another with answers. I designed a teacher spreadsheet that contained all the student data, together with calculations to determine key values and display appropriate graphs. Examples of these teacher spreadsheets can be found in MAV StartPoint Activities for 2013 and 2016 (among others).

SUMMARY

There is a wide range of material that can be used in many ways to set a challenging SAC activity for students in the political arena. The references provided are a starting point for whichever direction you choose to take. SAC tasks can be based on the Trial Paper questions, or the alternatives I have suggested in this article.

The key software used in the development of the questions was a spreadsheet (Excel) for data selection and analysis, and a graphic calculator computer simulator (TI-Nspire) to replicate student activities.

REFERENCES

Q1: *Wikipedia: List of Prime Ministers of Australia and List of Prime Ministers of New Zealand.*

Q2: https://www.aec.gov.au/profiles/

Q3: *Wikipedia: List of legislatures by number of members, List of United States state legislatures, List of states and territories of the United States by population.*

https://ballotpedia.org/Population_represented_by_state_legislators

https://info.australia.gov.au/about-government/states-territories-and-local-government/state-parliaments-and-members-contacts

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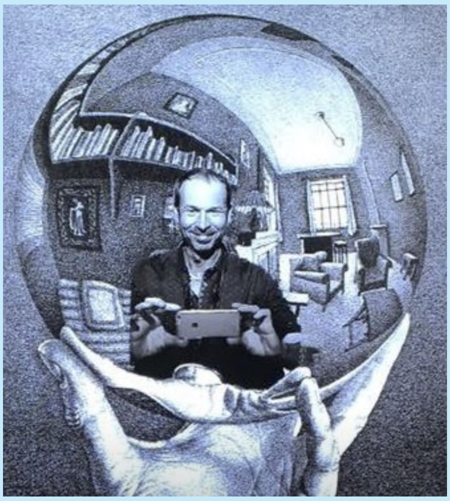
ONE MINUTE WITH JAMES TANTON



I'M...

James Tanton. I am an Australian fellow living in the US who really likes mathematics and I spend my days doing all that I can to promote the joy of mathematics. I have the wonderful title of Mathematician-at-Large for the Mathematical Association of America. I founded the Global Math Project, which is a grassroots initiative to transform the world's perception of what mathematics can be.

MATHEMATICS IS......

A beautifully human enterprise that has engaged humankind for many thousands of years. We humans continually discover and use mathematics, and we continually enjoy the play of mathematics. As such, each and every human on this planet should be properly invited to take part in its wonder.

I LOVE TO.....

Nut out a complicated piece of mathematics, or a piece of curriculum mathematics that is typically presented in a non-joyous way, and find the natural kernel of human insight that makes it all click.

I love teaching mathematics. I started my career in the research world (PhD in mathematics from Princeton) but followed my passion for teaching and outreach. I taught high school mathematics for ten years in the Boston area. I remember being required then to teach 16-year-olds how to divide polynomials. 'Why?' I wondered. As a research mathematician, did I ever divide polynomials? Not that I could recall. Will these students ever need to know this? Unlikely! After some mulling, I came to realise that polynomial arithmetic is simply the story of place-value in all bases, all at once. We humans tend to focus on base 10 (because of the count of digits on our hands), yet computer science focuses on base 2 (using only switches that are either on or off) and Martians, with six fingers, focus on base 6. So, let's make this unit a series of lessons that tell the story of one of humankind's (and Martian's) greatest intellectual achievements: the invention of place value.

My teenage students, of course, knew place value and could 'do' place value, but they were never given the opportunity to pause and reflect on the power of place value. By uniting the human story of working in base 10 (and 20, and 12, and 60, actually) with the universal stance of working in a general base x, suddenly all the mathematics I was required to teach was seen as a natural and easy repeat of primary school arithmetic, but from a stance of new intellectual power.

Why teach polynomial division? Because it provides a stunning opportunity for metacognition and power, and because it tells the story of great achievement of humankind.

Side note: This approach to sharing the story of place value and how it unites so much of the mathematics of the K-12 experience (and beyond!) has become a bit of a global phenomenon. Over 6.5 million students and teachers have played with the Exploding Dots story through the work of the Global Math Project.

A GREAT EDUCATOR IS.....

Able to fully recall and embrace and revel in not knowing and not understanding a concept at hand. The goal of a teacher is not to be an expert in content and to have all the answers but, instead to beautifully model for students the challenge of not knowing something and taking fumbling and flailing steps to move forward nonetheless.

Let's face it: all the content we teach is available on the internet. Teaching content cannot be the goal. Instead let's teach our next generations how to process content and also the confidence to just do something when faced with a challenge offering no clear steps forward.

I firmly believe that curriculum mathematics — high school mathematics — provides the perfect vehicle for teaching thinking and student agency.

I'M DELIGHTED BY....

Witnessing those beautiful 'aha' moments when a learner gets it and has that rush of delight and joy and enthusiasm for more!

STUDENTS DO BEST WHEN.....

The mathematics presented is honest.

THE LAST BOOK I READ WAS.....

The Data Detective by Tim Harford. This book was released under a different name in Australia (*How To Make The World Add Up*). Check out his *Cautionary Tales* podcast.

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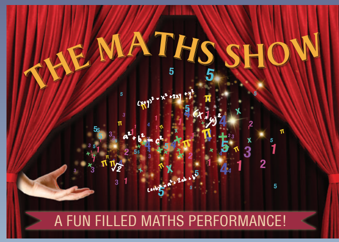

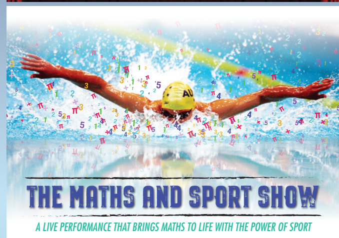

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CRITICAL AND CREATIVE THINKING

Dr Sharyn Livy – Monash University

ELICITING CRITICAL AND CREATIVE THINKING THROUGH THE USE OF SEQUENCES OF CONNECTED CUMULATIVE CHALLENGING TASKS

All students are capable of achieving in mathematics and making sense of important mathematical ideas. They learn by interacting with each other as their teacher uses evidence of student thinking to support and extend learning (NCTM, 2014). When developing students' skills of Critical and Creative Thinking teachers guide learning by providing mathematical experiences with breadth and depth as well as encouraging problem solving, reasoning and imagination (ACARA, 2021).

For the past three years I have been working with colleagues and primary school partners from Catholic Education Diocese of Parramatta and Melbourne Archdiocese Catholic Schools. This work is part of an Australian Research Council (ARC) Linkage project (LP 180100611) titled, Exploring Mathematics Sequences of Connected, Cumulative and Challenging (EMC^3) tasks.

My research includes investigating how Year 2 teachers can support their students to build new understandings in which mathematics is learnt when teaching sequences of challenging tasks. I have collected student data including work samples as evidence of children's ability to problem solve, reason and elicit Critical and Creative Thinking when responding to challenging tasks.

> Being able to use mathematical thinking in solving problems is one of the most fundamental goals of teaching mathematics … we want students to be able to conduct mathematical investigations by themselves (Stacey, 2006 p.39)

Therefore, teachers in the Early Years, and beyond, must be responsible for supporting their students with mathematical thinking in problem solving. Others would add, that while solving problems students should also be encouraged to struggle, make mistakes, persist think deeply, as well as think critically and creatively, with the overarching aim being to help students to develop a positive mindset towards mathematical learning (ACARA, 2021; Boaler, 2016; Sullivan et al., 2020).

Teachers are responsible for guiding student-centred learning approaches in order to prepare children with foundational mathematical thinking skills and support positive dispositions towards their learning of mathematics. Teachers in our project support Critical and Creative Thinking skills by providing opportunities for their students to: Reflect on thinking and processes; Identify, explore and organise information and ideas; Analyse, synthesise and evaluate reasoning and procedures; Generate ideas, possibilities and actions (Figure 1). Critical and Creative Thinking can be integrated across the curriculum and is one of seven General Capabilities of the Australian Curriculum that (ACARA, 2021).

Figure 1 shows the four elements of Critical and Creative Thinking. Both critical thinking and creative thinking are interconnected and support student thinking and learning of mathematics.

Figure 1. Organising elements for critical and creative thinking (ACARA, 2021).

The ACARA (2021) learning continuum of Critical and Creative Thinking lists six levels from Foundation Year to Year 10. Next we will explore how a Year 2 teacher might develop students' mathematical thinking when solving problems, including the four elements of Critical and Creative Thinking. Typically, at the end of Year 2 students, will demonstrate the elements of Level 2 (ACARA, 2021).

INQUIRING – IDENTIFYING, EXPLORING AND ORGANISING INFORMATION AND IDEAS AND ELEMENTS

By providing students with problems we are providing them with something to think about (Liljedahl, 2021) and opportunities to create connections for deepening their learning (Boaler, 2016). The types of problems designed for the EMC^3 tasks are mostly open ended, having more than one solution, and can be solved using different strategies. They cater for a range of student ability levels and are described as low floor and high ceiling tasks. Essentially this means that everyone can attempt the task then work at their own level to find the solution(s).

The Cubby House task was posed to a class of Year 2 at the beginning of the year (Figures 2 and 3). First the teacher launches the lesson without telling the students how to begin the task. Students are encouraged to pose questions to clarify their understanding before commencing the task (ACARA, 2021). They read the task then spent 5-10 minutes working on the task by themselves as part of the Explore Phase of the lesson. The students gather information from the task as they organise and processes information (ACARA, 2021). After reading the task the teacher might ask the students to identify key words in the problem, such as 9 friends and interpret the meaning of mathematical terms such as under and inside. Students could close their eyes and imagine the 9 children inside and under the cubby. Doing so helps the children to organise their thinking as preparation for attempting the task.

GENERATING IDEAS, POSSIBILITIES AND ACTIONS

The role of the teacher, prior to the lesson, is to have anticipated student responses. We encourage teachers when planning for teaching to do the problem in as many ways as possible and discuss how their students might record their thinking. The teaching team prepares an enabling prompt for students who may find the task too difficult after attempting the main task; and an extending prompt designed to extend student thinking and learning of the main task (Sullivan et al., 2020). For example, change the number of students from nine to six students playing in the cubby house as an enabling prompt; and for an extending prompt the students might be asked to generalise and convince you if they have found all the possible solutions for nine students.

CRITICAL AND CREATIVE THINKING (CONT.)

Prompts are important for helping students to build on what they know so they can create new ideas and make mathematical connections related to the main task.

All students explore the task, first by themselves, depending on the task students might work together after first attempting the task by themselves. Next, the teacher selects student work samples to share as part of the Summarise Phase to guide student thinking and learning by focusing on the mathematical goal of the lesson. The Summarise Phase of the lesson may occur more than once during the lesson as students re-engage with the task and record more solutions or use different strategies. Learning is generated from what the students are doing and thinking as students are guided to make mathematical connections (ACARA, 2021).

The Year 2 teacher noticed that students were drawing stick figures when recording solutions and one student was using dots to represent figures. The student (Figure 2) shared their work sample with the class as part of the Summarise Phase of the lesson. The teacher posed questions to the class, focusing on guiding students to notice the dots as an efficient method for recording the number of children in the cubby.

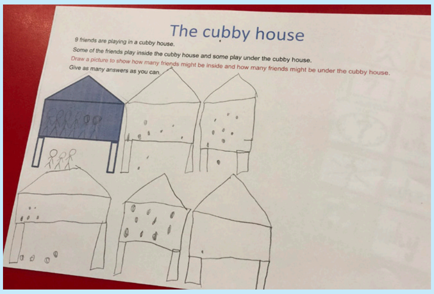

Figure 2. Student recording dots for the number of children in each cubby house.

Another student then recorded an efficient method for drawing the cubby house, which was shared and discussed with the class (Figure 3). In exploring both examples in Figures 2 and 3 the teacher was helping all students to reflect and consider ways to record their answers as they identified different combinations of nine children.

The teacher provided an opportunity for students to compare and discuss their diagrams, before seeking further solutions or all possible solutions.

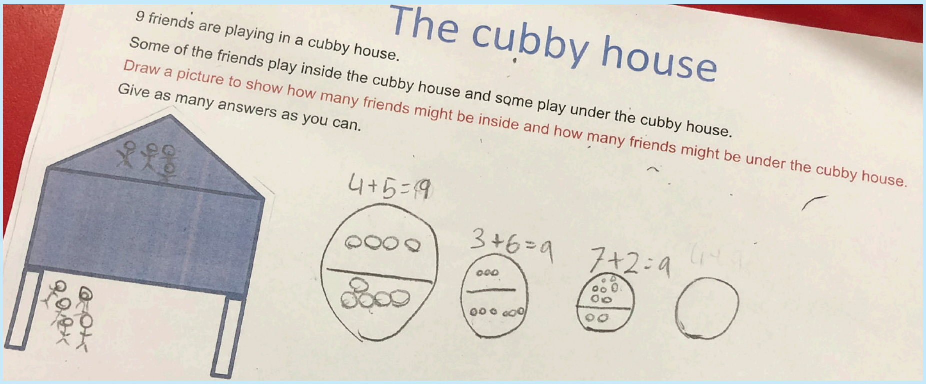

Figure 3: Student recording the cubby house with circles.

Figure 4: Student has recorded number sentences.

For students who had not attempted the task, such examples help students who may have been struggling.

REFLECTING ON THINKING AND PROCESSES ELEMENTS.

Focusing on the mathematics and working towards the number of possible solutions (extending prompt) the teacher drew students' attention to the recording of number sentences, discussing and sharing another work sample (Figure 4).

To help children to generalise and draw conclusions the teacher asks questions that assists student to explain and justify their thinking to highlight if they used patterns to find different answers. By sharing different responses to the task, the students can also reflect on each other's processes such as methods for finding different answers.

When thinking about thinking (metacognition) students describe their strategies (ACARA, 2021). One student said, 'I moved one student from the top of the cubby house to underneath,' to explain the processes of finding more solutions. Another student may have been reflecting on their processes when described their answers as, 'Rainbow (number) facts,' a term the students were familiar with when finding number facts for ten. As the students found one answer then another, they would also need to consider ways to check if there were more solutions. They could use information, each other's solutions, to inform their thinking and is evidence of transferring knowledge into new contexts (ACARA, 2021). Some might argue that these student actions also demonstrate analysing, synthesising and evaluating, the next element as students apply reasoning when thinking about their solutions.

ANALYSING, SYNTHESISING AND EVALUATING REASONING AND PROCEDURES

When students solve problems, they apply logic and reasoning; draw conclusions and evaluate whether they have found all the possible solutions (ACARA, 2021). Towards the end of the lesson, the teacher chooses a work sample that demonstrates particular outcomes such as exploring patterns and recording of most or all possible solutions. When discussing the work sample, the teacher has purposefully selected to guide the discussion, questions will be posed that help students to apply reasoning and thinking strategies.

Questions to the class or student sharing their work sample might include:

- Do you notice a pattern when recording your answers?
- Can you have the same number of children inside and under the cubby at the same time?
- How can you convince me that you have found all of the different answers?
- What is another way we might record the number sentence to check that we have all the answers?
- Can you record your answers in a table to help show your answers?
- What is your thinking in your head when adding 2 and 7?

We encourage teachers to plan questions prior to teaching to help student thinking when teaching the lesson, otherwise teachers will need to rely on in-the-moment decisions during instruction to guide students' thinking. Teachers can also plan and pose questions that assess students' understanding of the task and evaluate student reasoning.

Drawing on prior knowledge is evidence of drawing conclusions and designing a course of action (ACARA, 2021). The lesson approach and Variation theory used in the EMC^3 project provides students with opportunities to draw on prior knowledge and develop connected understanding of a mathematical concept. When solving tasks students can draw on prior knowledge to help analyse and synthesise a new problem (ACARA, 2021) and Variation theory assists with deeper understanding of a concept (Kullberg, et al., 2013).

When planning a sequence of lessons, either change the context of the problem or keep the same context and expand on the concepts. In other words, the tasks are designed so that students can draw on information from the previous task when exploring a new but similar task the following lesson. For example, the mathematical concept of the cubby house task helps children to develop part-whole thinking. Then the task, which the students will do the following day is varied:

> In a photo of a farmyard, you can see 12 legs; draw what the animals might be (give more than one possible photo).

The farmyard task connects with part-whole thinking, but the task is varied because the number has changed and is larger. When solving the 12 legs task, students may apply strategies such as efficient methods for drawing their solutions, look for patterns or record their responses in a table, connecting and building on their learning from the previous task.

The task reported here demonstrates how the lesson and teaching approach of teaching with tasks can support students' mathematical thinking and connected mathematical understanding. By using the teaching approach of anticipating student solutions, Launch, Explore and Summarise Phases of teaching with sequences of connected, cumulative and challenging tasks we anticipate that student in the early years will develop the mathematical skills and dispositions for investigating problems by themselves. Students as young as four and five should begin to draw conclusions, reason, make mathematical connections and develop critical and creative thinking skills. Importantly mathematics teachers of all year levels should be able to use tasks and teaching approaches that promote critical and creative thinking in their classrooms to help their students to use mathematical thinking when solving problems by themselves.

ACKNOWLEDGEMENTS

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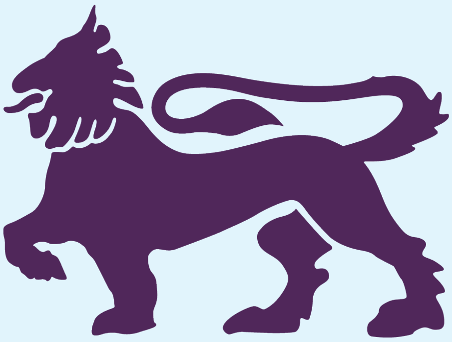
Dr Sharyn Livy will be exploring the topic of critical and creative thinking through the use of sequences of connected cumulative challenging tasks at MAV's 2021 annual conference. To learn more about the conference and to register, visit www.mav.vic.edu.au.

THE WESLEY COLLEGE LION

Lanella Sweet - Enrichment teacher, Wesley College

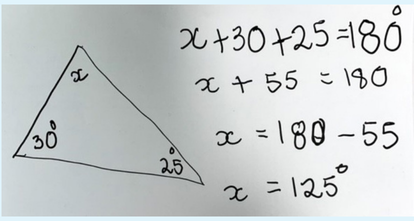


INVESTIGATIONS IN REMOTE LEARNING

This article will explore the possibilities of enriching knowledge and comprehension of a range of mathematical concepts in relation to a rich learning task focusing on the Wesley College lion.

In 2020, the Year 6 maths enrichment group explored a variety of mathematical concepts involving number, algebra, measurement and geometry.

During the Melbourne lockdown, whilst remote teaching took place, each skill was taught and practiced in isolation, ensuring all students could navigate the understandings of concepts, such as calculating areas of shapes and finding unknown angles within triangles and quadrilaterals.

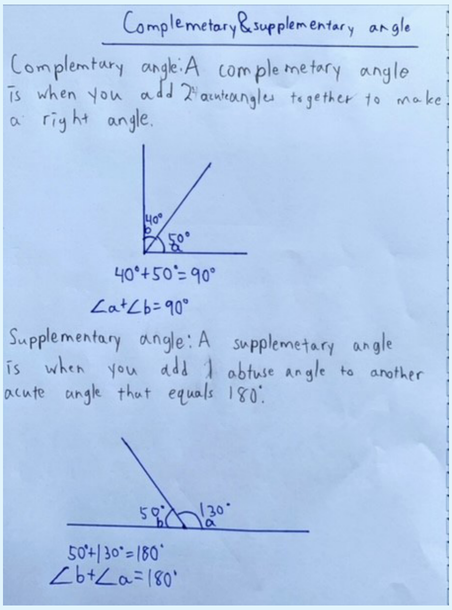


Once it was evident all students had solid understandings of processes and were confident in applying formulas, we began looking at how to calculate the areas of composite shapes. The students first identified and examined the two or more individual shapes forming the composite shape and then calculated and added up their areas. From this we made a collaborative decision to examine the specific irregular shape of our Wesley lion, the symbol of our school. Whilst students were missing face to face contact with staff and peers, this was an important way of bringing a little something from school into their homes.

Decisions were made by students and myself to make sure their own designs covered the mathematical concepts previously explored. Planning and creating ideas together enabled student's readiness levels and interest to match, which heightens motivation (Tomlinson, 2004).

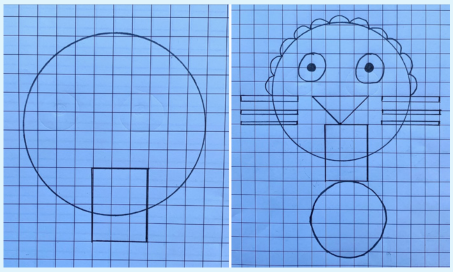
MATHEMATICAL CONCEPTS AND THE DESIGN OF THE LION

Students authentically moved towards the goal of creating their own design and drawing of the Wesley lion. A 'check list' was created to ensure students incorporated all the previously taught skills into their lion. Students needed to specifically include the calculation of area, perimeter and demonstration of various angles, classification of triangles, to name a few.



The students worked on designing their Wesley lion to demonstrate previously learnt content. Sometimes this required them to practise, refine and improve with feedback. Suitable technology was used to facilitate discussions and time to collaborate with peers and staff, allowing the students to improve with feedback. For example, some students only used simple shapes initially, however after time, these students could draw and calculate composite shapes made up of three or more smaller shapes.

Students modified their approach over time, when they developed more confidence in using skills such as displaying supplementary and complimentary angles.



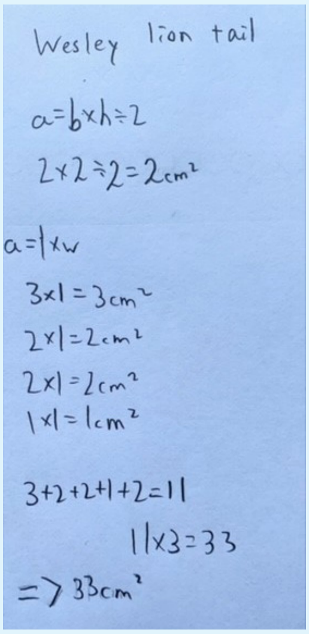
Students could then use application skills to demonstrate appropriate understanding. They became problem solvers with their own design to ensure they covered all the concepts from the checklist. Expert problem solvers constantly monitor their thinking, evaluating ideas, checking their progress and keeping their work in line with the original goal (Stacey, 1995). Students became very comfortable with the skill of talking through their reasoning and expanding on their thought processes, making modification(s) after feedback and collaborative discussions.

ESTABLISHING ACADEMIC GOALS AND OUTCOMES

Along with the checklist, the students and I created various guiding statements to assist with the task and to prompt broad, deeper thinking. Some of these collaborative statements are:

- Include at least one of each shape.
- You can show both quadrilateral shapes along with various triangular shapes.
- Look at the whole Wesley lion.
- Show all geometry skills learnt in class.
- Try to label diagrams, such as arc, reflex angle.
- Challenge yourself, with shapes overlapping other parts.
- Try to demonstrate your understanding of perimeter and parallel lines.
- Challenge yourself to show unknown values with algebraic expressions.
- Reinforce your understanding of complementary and supplementary angles.
- Try to use a variety of shapes to really demonstrate your mathematical knowledge and skills.
- Show details of how you calculated perimeter and area of every shape within the whole Wesley lion design.

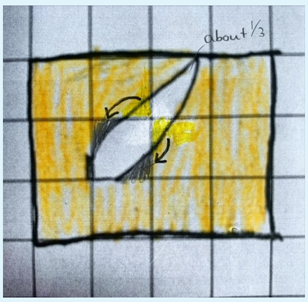
Students were given choice on presenting their findings. With these guidelines, some students chose to focus on particular, smaller areas of the lion, for example the face, while others focussed on larger, broader shapes for the outline of the lion.



Students were able to demonstrate their independent ideas and work at their appropriate level of development. Offering options and alternatives based on choice enriched the curriculum for those students (Tieso, 2004, 61).

Students dived deeply into the task and their findings were impressive. Some of the ideas explored, in addition to basic area and perimeter, are listed below, quite comprehensive for a single task.

- Calculating areas of irregular shapes and utilising negative space, as illustrated.



- Negative space is unused in the drawing, so the estimate of the two black areas would be subtracted from the area of the square (1 cm^2).

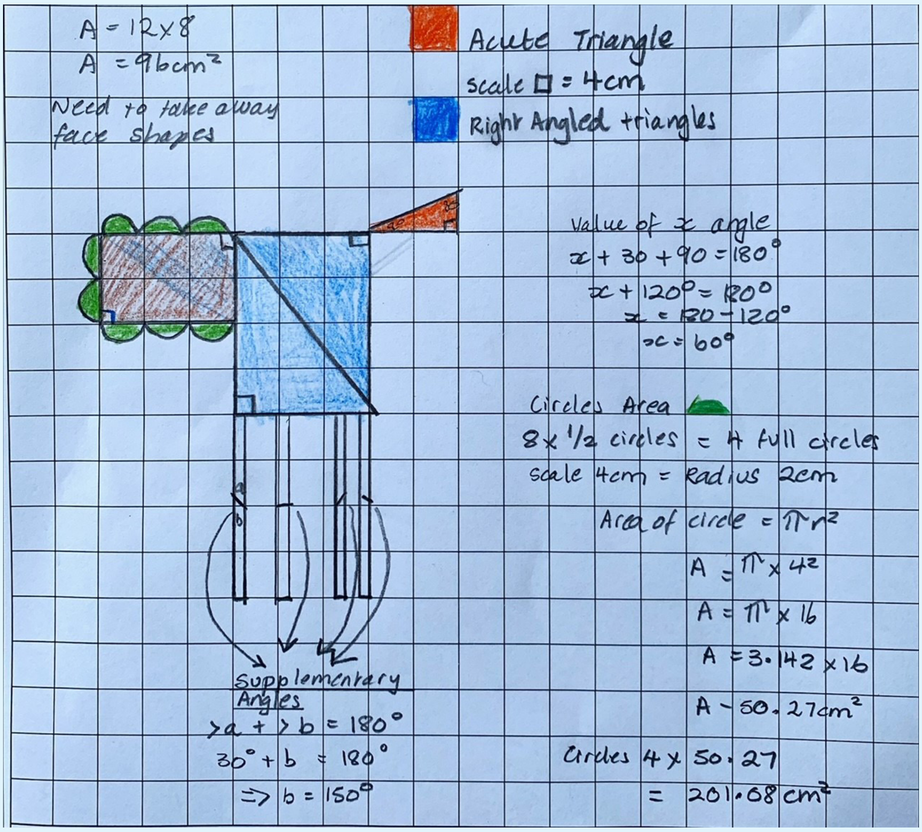


Figure 1.

- Examining and demonstrating application of algebra to triangles.
- Designing smaller parts of a shape to replicate a larger shape.
- Examining various shapes close up, for example incorporating the lion eye with a combination of triangles and rectangles.
- Calculating area and perimeter and looking at the correlation of the values (See Figure 1).
- Perspective and scale, including applications to areas (See Figure 2).
- Thinking mathematically, broadly and specifically at the details of design.

PROBLEM SOLVING TECHNIQUES

Many problem solving techniques were embedded into the investigation. Students moved fluidly through George Polya's four step model for problem solving:

1. Understanding the problem
2. Devising a plan
3. Carry out the plan
4. Looking back

In Polya's work, investigations into the mathematical heuristics lay the foundation for many further explorations and can be still be viewed as relevant in today's mathematical classroom. The students used knowledge obtained from previous attempts and trials to continue and find the most appropriate solution. For example, one student drew the lion face with very simple shapes, then with collaboration and feedback, added triangles, rectangles and circles appropriately and to scale to show further required understandings.

Polya's 'looking back' concept was extended on by Cai and Brook (2006) where three approaches within this final step were considered:

1. Generating, analysing, and comparing alternative solutions,
2. Posing new problems
3. Generalising

The lion investigation allowed students to 'look back' and further extend learning mathematical knowledge and acquire new abilities.

THE WESLEY COLLEGE LION (CONT.)

They had not thought about the bigger picture of combining mathematical skills to form something larger and broader. The students were able to articulate their new knowledge and transfer previous skills and also to make generalisations and pose new problems.

STUDENT'S INDEPENDENCE AND COLLABORATION WITH MATHEMATICAL THINKING

The lion investigation was open-ended and rich. It allowed students to think mathematically and work at their own pace. Students were asked to record their outcomes and collaborate with other members of the group, promoting the best method of peer tutoring, talking through thinking and group co-operation structures using suitable technology. Students enjoyed the independence of the group work; they felt empowered to try new communication methods as a result and felt a greater ownership over their learning. This engagement promotes multiple outcomes and solutions to enhance learning capacity and learning opportunities.

Students had some common characteristics:

- Spent time thinking before making decisions. I had thinking time announced to the group before any students actually put pen to paper.
- Identified the nature of the criteria and expectations and then organised work.
- Effectively drew on previous mathematical skills and knowledge.
- Used efficient computation and arithmetic skills.
- Used trial and error, developing further understanding and modifying their designs after each trial.
- Looked for and identified pattens and links between shapes, angles, perimeter and design.
- Demonstrated mathematical understanding in an appropriate manner.

Expert problem solvers constantly monitor their thinking, evaluating ideas, checking their progress and keeping their work in line with the original goal (Stacey, 1995).

As the students effectively understood concepts they were adequately challenged,

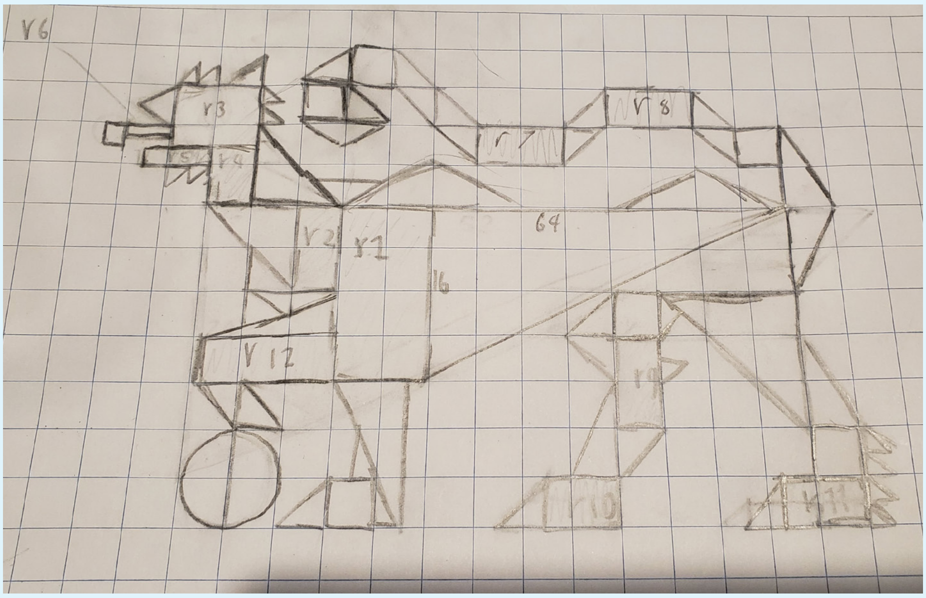


Figure 2. Perspective and scale.

which evoked a sense of motivation and therefore increased learning engagement. The art of problem-solving is the heart and essence of mathematics, because problem-solving can serve as a vehicle for learning new mathematical ideas and skills (Perven, 2010, p10).

STUDENT REFLECTIONS

Matthew: The lion investigation gives you the opportunity to break a common image into smaller parts and then find the true value of that image (mathematically). I like converting what I see into angles and shapes to form the Wesley lion and apply my maths knowledge about measurement and algebra. The creating of the picture is fun, and you still get to calculate and think mathematically about things like area and use maths formulas which is quite enjoyable – two good things in one.

Campbell: I love learning new things and then being able to apply my knowledge into something that I like, and I look forward to every maths session.

Fahiz: We can now apply maths using my knowledge of triangles, shapes, angles and area, perimeter, which we have learnt. Applying this is a fun way whilst being able to understand all the things we have learnt in one.

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